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Clustering and synchronization of lightning flashes in adjacent thunderstorm cells from lightning location networks data

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[1] We analyzed sequences of lightning flashes in several thunderstorms on the basis of data from various ground-based lightning location systems. We identified patterns of clustering and synchronicity of flashes in separate thunderstorm cells, distanced by tens to hundreds of kilometers from each other. This is in-line with our early findings of lightning synchronicity based on space shuttle images (Yair et al., 2006), hinting at a possible mutual electromagnetic coupling of remote thunderstorms. We developed a theoretical model that is based on the leaky integrate-and-fire concept commonly used in models of neural activity, in order to simulate the flashing behavior of a coupled network of thunderstorm cells. In this type of network, the intensity of the electric field E_i within a specific region of thunderstorm (i) grows with time until it reaches the critical breakdown value and generates a lightning flash while its electric field drops to zero, simultaneously adding a delta E to the intensity of the internal electric field in all thundercloud cells ($E_{j,k,l}$. . .) that are linked to it. The value of ΔE is inversely proportional to the distance between the "firing" cell i and its neighbors j, k, l; we assumed that thunderstorm cells are not identical and occupy a grid with random spacing and organization. Several topologies of the thunderstorm network were tested with varying degrees of coupling, assuming a predetermined probability of links between active cells. The results suggest that when the group coupling in the network is higher than a certain threshold value, all thunderstorm cells will flash in a synchronized manner.

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1. Introduction

1.1. Onset of Lightning Discharges

[2] The observed occurrence of lightning flashes within a thunderstorm reflects the repeated exceeding of the breakdown electric field in several parts of the storm. Every single flash is essentially the culmination of numerous dynamical and microphysical processes which operate synergistically to separate electrical charges and build an electric field within the cloud. While the general features of lightning activity along the cloud life cycle have been known for quite sometime [*Mason*, 1953], the exact nature of the microphysical charge separation processes which amplify the electric field remains elusive, even after years of observations, laboratory studies, and modeling (see reviews by *Jayaratne* [2003] and *Rakov and Uman* [2003]). The lightning flash rate in different thunderstorms differs considerably. Winter thunderstorms, like those occurring in Japan and the eastern Mediterranean, typically exhibit low rates of several flashes per minute. On the other hand, summertime mesoscale convective systems can produce flash rates of several tens per minute, with some cases exceeding 200 flashes per minute. Such high flash rates are a clear manifestation of the potency of charge separation mechanisms operating within these thunderclouds which often display vigorous convection, a large vertical dimension and high water contents.

[3] The inception of lightning is believed to occur in regions where the concentrations of ice particles, graupel pellets and supercooled liquid water is highest. However, the exact nature of the onset of the breakdown process is not well understood. Various mechanisms which have been suggested, such as the initiation of streamer discharges by a chain of water drops [*Nguyen and Michnowski*, 1996], the production of MeV runaway electron avalanches initiated by cosmic rays [*Gurevich et al.*, 1992, 1999], or the generation of positive streamers at the surface of hydrometeor particles [*Griffiths and Phelps*, 1976]. We refer the interested reader to the extensive reviews by *Rakov and Uman* [2003] and *Cooray* [2003a] for further reading. The suggested

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processes may indeed play a role in generating streamer discharges, but for these to propagate and become lightning leaders developing to a full discharge a critical ambient electric field is required. The measured intensity of electric fields within thunderclouds seldom exceeds 150 kV/m [*Marshall et al.*, 1995] and is much lower that the conventional breakdown field for air at the altitudes of charge centers in clouds (~5 km), which is of the order of 1 MV/m. Such values are within the magnitude of the "breakeven" field which must be exceeded for runaway breakdown to propagate [*Suszcynsky et al.*, 1996], a mechanism further supported by recent observations of X rays in thunderstorms [*Dwyer et al.*, 2003] and by correlations found between the soft component of cosmic rays with the local electric field during thunderstorm periods [*Khaerdinov et al.*, 2005].

1.2. Observations of Lightning Synchronicity

[4] While an enormous amount of research was devoted to the initiation of a single lighting flash, the characteristics and properties of the sequence of consecutive flashes received little attention, most likely owing to the inherent assumption that a lighting flash is a stochastic process, unrelated and independent of the occurrences of other flashes in the cloud. *Dennis* [1970] visually recorded lightning in 20 New Mexico thunderstorms and conducted a statistical analysis of the timing of consecutive flashes. The results show that the logarithms of the time intervals between successive flashes within a given storm are distributed normally, with a median flash interval between 24 and 290 s. The computed autocorrelation coefficient over all possible time lags was in the range 0–0.2, which led *Dennis* [1970, pp. 170–172] to conclude that "intervals between flashes throughout a given storm are essentially independent of one another." A limited test for a multiple-cell storm was conducted using radar with similar conclusions. The low values of autocorrelations and lack of detailed information on the charge distribution within the cloud led *Dennis* [1970, pp. 170–172] to finally conclude that "the occurrence of an individual flash must be considered a random phenomenon" and that "little meteorological significance can be attached to details of the time distribution of the flashes in a particular storm." It is important to note the poor accuracy of the manually registered interarrival times which was 1 s, but in many cases was poorer since times were rounded to the nearest 5 s. The daytime observations were solely of cloud-to-ground flashes while the nocturnal ones were based on any illumination of the observed cloud. Thus the reliability of the observations and the limited amount of analyzed cases suggest that there may be a different way for interpreting interarrival times of flashes. In a different study by *Mazur* [1982] using VHF radar echoes of lightning in different "electrically active cells" (EACs), many cases were reported of "associated discharges," namely, flashes occurring within a 200 ms time interval from one another, typical for multistroke cloud-to-ground flashes. Mazur studied the null hypothesis that all observed flashes were independent renewal events with normal distributions of interarrival times. By employing a statistical analysis on three 1-min samples of lightning data (each containing consecutive flashes) he showed that the null hypothesis is rejected at levels of significance between 0.1% and 5%. The physical interpretation of this interdependence was the electrical interaction between neighboring EACs in a multicell storm, where the collapse of an electric dipole in one cell (due to lightning) induces a sudden change in the E field vector in the neighboring cell. The magnitude and timing of this change may speed or delay the next flash in the cell, and "associated discharges" are a manifestation of this shortening of the time intervals.

[5] A similar phenomenon of closely spaced flashes was reported by *Yonnegut et al.* [1985], who noted the clustering or convergence of lightning flashes as they appeared in nocturnal space shuttle video images. They offered a conceptual explanation similar to that of Mazur, stating that the release of electrical energy in one portion of the cloud eventually triggers the development of a new breakdown process in another part. In another study using space shuttle lightning images, *Yair et al.* [2006] analyzed footage from six storm systems observed during the MEIDEX [*Yair et al.*, 2004] and showed that in storms exhibiting a high flash rate, lightning activity in separate electrically active cells showed transient synchronization with bursts of nearly simultaneous flashes in different cells. They proposed that this behavior is equivalent to the collective dynamics of a network of weakly coupled limit-cycle oscillators. According to this hypothesis, thunderstorm cells embedded within a mesoscale convective system (MCS) constitute a network, and the flash occurrence rate can be described in terms of phase locking of a globally coupled array. Comparison of basic parameters of the observed lightning networks with predictions of random-graph models revealed that they are best described by generalized random graphs with a prescribed degree distribution [*Newman et al.*, 2001], which typify networks supporting fast response, synchronization, and coherent oscillations.

[6] Most recently, *Ondrášková et al.* [2008] reported on peculiar transient events in the Schumann resonance band of the extremely low frequency range (ELF, 0.1–20 Hz), consisting of two overlapping signals with typical differences of 0.13–0.15 s between their onset times. These signals are attributed to lightning flashes occurring almost simultaneously in the same thunderstorm, which coherently excited the Earth-ionosphere cavity. A similar phenomenon in the ELF range, albeit with a ~2 s delay, was reported by *Füllekrug* [1995].

1.3. Tests of Randomness for Time Series of Lightning Flashes

[7] Remarkably, there is little work conducted on the distribution of the times of consecutive lightning flashes in a given storm even though lightning location systems with accurate GPS time stamping, like the U.S. National Lightning Detection Network and the French SAFIR system, are fully operational since the mid-1980s [*Orville*, 2008]. While the interstroke timing statistics are well established and robust [see *Rakov and Uman*, 2003, chap. 4], not much was published on the statistics of the interarrival times of individual flashes. In order to investigate this we utilized lightning data from several different detection systems and studied the occurrence sequence of lightning flashes. We will demonstrate the method for a 1 h of lightning activity in a storm that took place near the coast of Lebanon on

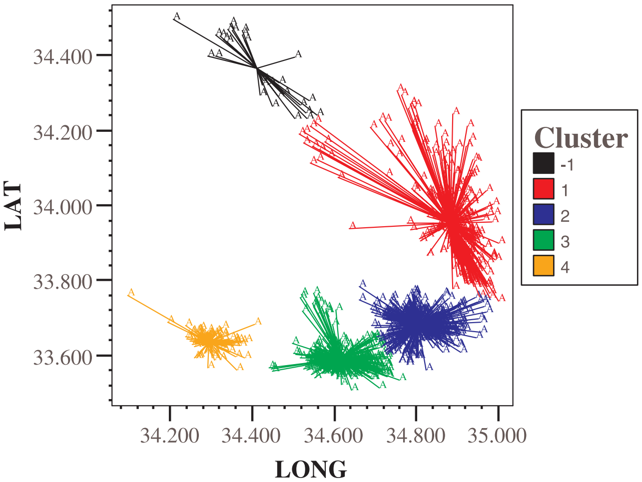

Figure 1. Clusters of lightning events in 1 h of data from the storm that occurred on 29 October 2004 between Cyprus and Lebanon. The data are based on Lightning Positioning and Tracking System registrations of cloud-to-ground flashes.

29.10.2004. The data was taken from the Israeli Lightning Positioning and Tracking System (LPATS) which registers cloud-to-ground flashes in a range ~600 km from the coast of Israel. Since the LPATS, as other lightning location systems, is capable of resolving separate strokes which comprise a single flash, it is necessary to group subsequent strokes into a well-defined flash before the analysis so that it can be attributed to a given cell. In the present analysis we only used the flash data, where the first stroke represented the discharge. It should also be pointed out that in the present work we refer to each thundercloud as containing several charged cells, which are separated physically and whose state is a function of the microphysical conditions within the storm.

[8] To identify the active cells we used SPSS two-steps cluster analysis. Clustering was based on the distances between the actual locations of the lightening events. The number of clusters was automatically determined by the software according to the Schwartz Bayesian criterion (BIC). As an example, Figure 1 presents the results for 1 h of data. We see that the lightning events are clustered into four cells that are well separated from each other by tens of kilometers. (Cell −1 contains few "outliers" events that could not be assigned to any cell.) The distribution of all of the 1256 events among the clusters is presented in Table 1. An independent consistency check for that storm was performed using Matlab. First we calculated the local densities of lightning events (events/km^2) and then drew equal-flash densities contours on the geographic map of the area, where different density levels were coded by different colors (Figure 2). Visual inspection clearly identified the same four well-separated lighting clusters.

[9] The distributions of interarrival times between consecutive lightning flashes in the Lebanon storm system are highly positively skewed. To illustrate this, we present in Figure 3 the distribution of the interarrival times between consecutive flashes for the entire system. The results in Table 2 present the sequence of lightning events from each parent cell, designated A–D. If we define a time scale of 0.2 s or less as the threshold, which reflects synchronous flashes [*Mazur*, 1982], the results show that there were 401 events (out of 1226) that fall within this time frame. In the vast majority of cases (94%) a flash in a given cell was preceded by a flash in the same cell. If the threshold between consecutive flashes is set at 1 s, then out of 677 events, 90% originate from the same cell. This may suggest that the collapse of the electric field in a given region of the cloud is somehow transmitted to adjacent cells and leads to the rapid onset of breakdown there.

[10] The associated descriptive statistics for sequences of flashes from each of the cells as well as the complete system are presented in Table 3. The medians of the interarrival times range from 0.12 to 0.53 s, but the 75th percentiles are larger by an order of magnitude. From Figure 3 we see that the distributions of the interarrival times in the complete system deviate considerably from an exponential distribution (note the logarithmic scale of the vertical axis). In calculating the exponential distribution we used the actual average interarrival time taken from the data. An important point is that the deviation is at the low end of the distribution. Thus, lightning flashes tend to cluster and occur in close succession.

[11] To formally test whether the process generating these lightning events is random, we applied the Kolmogorov-Smirnov (K-S) test. Specifically, we tested whether the distribution of interarrival times is exponential (which it should be, if the lightening events were generated by independent Poisson processes). We used both asymptotic K-S as well as Monte Carlo simulation with 50,000 samples. Details of the tests and results are presented in Table 4. The value 0 for Sig indicates that for all of the four cells (lightning clusters) the hypothesis that the distributions of the interarrival times of lightning flashes is exponential is rejected at the 99% confidence level; thus it is highly improbable that the lightning events were generated by an independent Poisson process. Repeated tests for other lighting sequences in different storms gave essentially the same results.

[12] We conclude that the sequence of lightning flashes in a given thunderstorm cell as well in the four-cell system is not random; there is a tendency of interarrival times to have values that are significantly shorter than the prediction of the exponential distribution. This convergence phenomenon points to the possibility that generation of lighting flashes within and between cells are somehow synchronized. We now seek to decipher this behavior in terms of network dynamics.

2. Adaptive Oscillator Network Model of Electrically Active Cells

[13] A network is a system composed of several separate interacting entities, relating to each other in different modes with varying levels of complexity. There is now a vast body of theoretical and experimental research that relates the properties of complex dynamical phenomena in nature to those described by network theory [*Strogatz*, 2001]. We assume that thunderstorms developing in a synoptic-scale or mesoscale weather system comprise a network of leaky integrate-and-fire oscillators (LIF), loosely affecting one another by long-range electromagnetic processes. Every thundercloud may posses several such oscillators, each

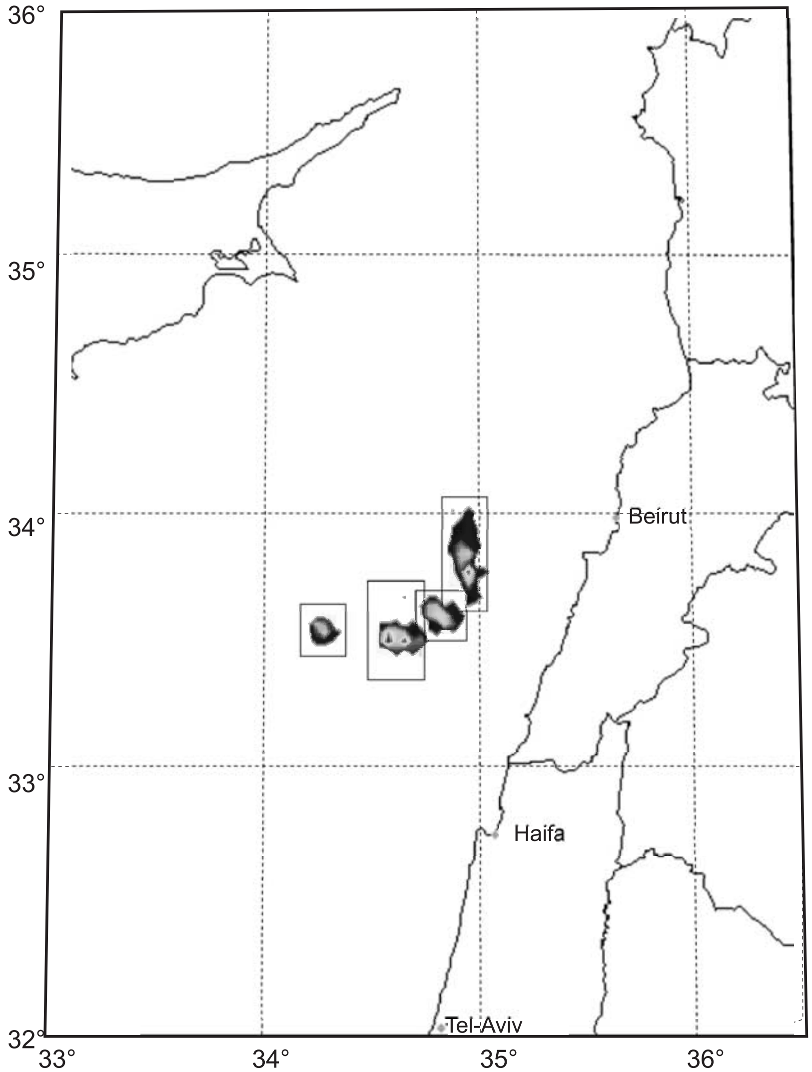

Figure 2. Flash density derived from the Israeli Electrical Company lightning detection network for 1 h of lightning activity, for the storm of 29 October 2004. Orange and yellow colors denote higher values.

representing a highly charged region with the potential for lightning discharge. The term LIF is borrowed from works on the electrical activity of neurons, where the pulsing behavior of networks of connected nerve cells is often simulated [*Shimokawa et al.*, 1999; *Chacron et al.*, 2004]. The analog to a breakdown of the cloud electric field and the subsequent lightning discharge is the spiking of the membrane potential of a nerve cell. A biological leaky integrate-and-fire oscillator

Table 1. Distribution of Lightning Events Among Active Cells, or Clusters

Cluster	N	Percent of Total
1	332	26.5
2	408	32.5
3	376	29.9
4	110	8.8
Outlier (−1)	29	2.3
Combined	1256	100.0
Total	1256	100.0

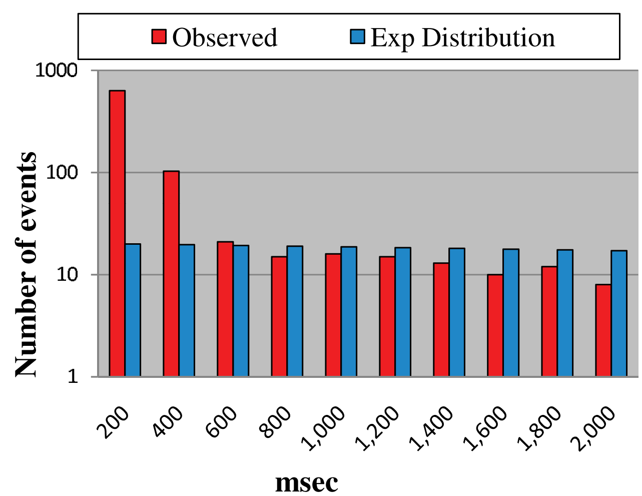

Figure 3. Histograms of the interarrival times for the Lebanon storm.

Table 2. Lightning Numbers Derived From the Flash Sequence of the Lebanon Case Study[a]

	A	B	C	D	Total
A	199	59	56	18	332
B	61	278	60	9	408
C	56	60	244	16	376
D	16	11	16	67	110

[a]We classify the lightning events into 16 classes, according to the cell where the event occurs and the cell in which the preceding event occurred.

is typically charged by a given process until it reaches a certain critical threshold level, above which it fires a pulse and goes back to its initial state. In electricity parlance, such an oscillator consists of a capacitor C in parallel with a resistor R driven by a charging current I(t). Systems of identical biological oscillators have been studied extensively in relation to pulse coupling and synchronization [*Mirollo and Strogatz*, 1990]. That work showed that in almost all initial conditions, the system evolves to a state where all oscillators are firing simultaneously. The rapid development of the study of complex networks [*Strogatz*, 2001; *Hong et al.*, 2004] was followed by introduction of new types of networks being investigated for synchronization. Such are for example adaptive networks, where rewiring of the network structure is done in order to enhance mutual synchronization [*Gleiser and Zanette*, 2006]. We decided to adopt such an approach to study the individual and group behavior of thunderstorms. Thus, we model thunderclouds as ensembles of electrically active cells, each represented by several oscillators whose dimensionless intensity $X_i(t)$ is subject to the differential equation

$$dX_i(t)/dt = -\gamma_i X_i + I_i + S_i(t). \qquad (1)$$

$X_i(t)$ is analogous to the electric field in a specific region of the thundercloud, which when passing its maximum, leads to the generation of a lightning flash. This maximum value is assumed to be identical to all cells in all clouds and normalized to 1, considering the fact that breakdown in clouds mostly takes place in a well defined pressure range (e.g., altitude) and is essentially the same. The oscillator is "charged" by the constant "current" I_i, but its intensity leaks; the leakage rate factor is γ_i and $I_i > \gamma_i$. The term $S_i(t)$ represents the contribution to the rate of change of the intensity $X_i(t)$ obtained from the interactions with other neighboring EACs. We shall first look at identical noninteracting EACs, then incorporate interactions with other EACs, and finally introduce heterogeneity into the system, to better represent the natural variance of thunderstorms.

2.1. Noninteracting LIF Oscillators

[14] The solution of equation (1) with no interaction (namely, when $S_i(t) = 0$), restricted to be between 0 and 1, is the periodic function with a period T_i:

$$X_i(t) = (I_i/\gamma_i) * (1 - e^{-\gamma_i t}) \text{ for } 0 < t <= T, X_i(t+T) = X_i(t), \qquad (2)$$

where T_i is given by

$$T_i = (1/\gamma_i) * \ln(1 - \gamma_i/I_i)^{-1}. \qquad (3)$$

The oscillator gets the threshold value 1 at the time $t_{i,m} = mT_i$, m = 1, 2, . . .

[15] It is convenient to replace the time variable by the phase of the oscillator: the phase of an oscillator i, $\varphi_i(t)$, is linearly rising function of time with slope $1/T_i$, that is $d\varphi_i/dt = 1/T$ for all values of t except for the values $t = t_{i,m}$. At these points $X_i(t_{i,m}) = \varphi_i(t_{i,m}) = 1$ and $\varphi_i(t)$ is discontinuous: φ_I jumps to 0, that is $\varphi_i(t_{i,m}) = 0$, and it starts rising again.

[16] In the case of no interaction $\varphi_i(t) = t/T_i$ for $0 < = t < = T_i$. The intensity of the oscillator is then given by the function

$$X_i = f_i(\varphi) \equiv \left(1 - e^{-\gamma_i T_i \varphi}\right)/\left(1 - e^{-\gamma_i T_i}\right) \text{ for } 0 < \varphi <= 1, \qquad (4)$$

where the phase function is given by

$$\begin{array}{llll} \varphi(t) = (1/T_i)*(t - t_0) & \text{for} & t_0 < t <= t_0 + T_i \\ \varphi((t_0 + T_i)+) = 0 & \text{for} & \varphi(t + MT_i) = \varphi(t) \end{array} \qquad (5)$$

for all M integers. t_0 is the time at which the oscillator started to live. This value varies between oscillators. The function $f_i(\varphi)$ determines the intensity of oscillator i in terms of its phase. The inverse function, that determines the phase given the intensity is $\varphi = g_i(X)$, is

$$\varphi = g_i(X) \equiv (1/(\gamma_i T_i)) * \ln[1 - (1 - \exp(-\gamma_i T_i))X]^{-1}. \qquad (6)$$

2.2. Interacting Oscillators

[17] We assume that an EAC i is influenced by a certain subset of the oscillators, the influencing neighbors, denoted

Table 3. Statistical Parameters of the Distribution of Interarrival Times in Seconds for Lightning Flashes in the Lebanon Storm[a]

	Min	Max	Prcnt25%	Median	Prcnt75%	Average
Cell 1	0.0150	135.3740	0.0790	0.5350	16.7125	10.7972
Cell 2	0.0150	212.2780	0.0620	0.1740	11.9455	8.6781
Cell 3	0.0140	126.0880	0.0605	0.1930	14.6830	9.3330
Cell 4	0.0150	1120.1350	0.0500	0.1230	24.8230	18.202
All cells	0.0140	1120.1350	0.0650	0.2270	16.3430	11.9294

[a]Prcnt25% and Prcnt75% denote the 25th and 75th percentiles, respectively.

Table 4. One-Sample Kolmogorov-Smirnov Test

Parameter	Value
Cluster 1	
N	332
Exponential parameter (s)	10.797
Kolmogorov-Smirnov Z	8.340
Asymptotic Sig (two-tailed)	0.000
Monte Carlo Sig (two-tailed)	0.000
Cluster 2	
N	407
Exponential parameter (s)	8.678
Kolmogorov-Smirnov Z	11.185
Asymptotic Sig (two-tailed)	0.000
Monte Carlo Sig (two-tailed)	0.000
Cluster 3	
N	375
Exponential parameter (s)	9.332
Kolmogorov-Smirnov Z	10.499
Asymptotic Sig (two-tailed)	0.000
Monte Carlo Sig (two-tailed)	0.000
Cluster 4	
N	108
Exponential parameter (s)	18.202
Kolmogorov-Smirnov Z	6.079
Asymptotic Sig (two-tailed)	0.000
Monte Carlo Sig (two-tailed)	0.000

by Pre(i); in turn, it influences EACs in the subset Post(i) of influenced neighbors. The influence on EAC i is

$$S_i(t) = \Sigma_j \; \Sigma_m \; \varepsilon_{ij}(t) * \delta(t - t_{j,m} - \tau)) \quad \text{j belong to Pre(i)m} = -\infty \text{ to } +\infty. \tag{7}$$

$S_i(t)$ is the contribution to the change in the intensity due to signals sent from oscillators in Pre(i), which arrive to oscillator i at time t. Specifically, a neighbor oscillator j emits a signal at times $t_{j,m}$ when it gets its upper limit, $X_j(t_{j,m}) = 1$; at time $t = t_{j,m} + \tau$ this signal arrives to i. τ is the signal delay time. As a result the intensity of i is incremented by an amount $\varepsilon_{ij}(t) = \varepsilon_{ij}(t_{j,m} + \tau)$.

[18] In this model, the oscillators are free (noninteracting) except for the times of events of signal receptions. Hence, between these events the phases of the oscillators increase linearly, $\Delta\varphi_i = \Delta t/T_i$ and the relations between the phases and the intensities are given by equations (4) and (6). When an oscillator receives a signal its phase jumps. Specifically, if φ and $f(\varphi)$ are the phase and the intensity of an oscillator i just before receiving a signal, then its intensity jumps to $f_i(\varphi) + \varepsilon$, and hence its phase jumps to $h_i(\varepsilon, \varphi) = g_i(f_i(\varphi) + \varepsilon))$. This is called the transfer function. For a LIF-type oscillator we get

$$h_i(\varepsilon, \varphi) \equiv (1/(\gamma_i T_i)) * \ln[1 - (1 - \exp(-\gamma_i T_i))(f_i(\varphi) + \varepsilon)]^{-1} \tag{8a}$$

and hence

$$h_i(\varepsilon, \varphi) \equiv 1/(\gamma_i T_i) * \ln[\exp(-\gamma_i T_i \varphi) - (1 - \exp(-\gamma_i T_i))\varepsilon]^{-1}. \tag{8b}$$

Now consider a signal sent from oscillator j who belongs to Pre(i), at time t'. Oscillator i received it at $t = t' + \tau$, where its phase and intensity before receiving the signal are $\varphi_i(t)$, and $f_i(\varphi_i(t))$ respectively. After receiving the signal the intensity jumps to $f_i(\varphi_i(t)) + \varepsilon_{ij}$. If this new intensity is below 1, the new phase is $h(\varepsilon_{ij}, \varphi_i(t))$; otherwise oscillator i sends a signal and its new phase and intensity reset to 0. This is summarized as

$$\varphi_i((t)^+) = h(\varepsilon_{ij}, \varphi_i(t)) \text{ if } f_i(\varphi_i(t)) + \varepsilon_{ij} < 1, \tag{9a}$$

$$\varphi_i((t)^+) = 0 \text{ if } f_i(\varphi_i(t)) + \varepsilon_{ij} \geq 1. \tag{9b}$$

In this work we assume that ε_{ij} is inversely proportional to the distance between the oscillators, that is $\varepsilon_{ij} = \varepsilon/Rij$. Here ε is a global coupling constant. $R_{ij} = \text{sqrt}[(x_i - x_j)^2 + (y_i - y_j)^2]$ where the integers (x_i, y_i) are the coordinates of the oscillator i in a two-dimensional grid.

2.3. Adaptivity

[19] Suppose that a signal is sent from oscillator i. After a short delay τ, the signal will reach the oscillators belonging to the set Post(i), and if ε is large enough then all members of this set will immediately reach their threshold and fire new signals. After another short time interval τ these signals will reach their Post sets, their members will jump to their threshold level, firing new signals. This process will repeat itself. Thus, an avalanche of signals will occur during a short interval. In the limit of strong coupling and no delay ($\tau = 0$) all oscillators will continue to fire signals simultaneously and repeatedly. In this limit the oscillators are strongly synchronized. With a nonzero τ but still large coupling, signals will arrive to the Post sets after some delay, so that groups of oscillators will fire in unison. If all the oscillators are directly connected to each other then the Post set of any oscillator consists of all other oscillators. In that case all the oscillators will fire in unison, providing again a completely synchronized system. The phases of all the oscillators periodically converge to 0. When this is not the case, we will have partial synchronization: various groups of oscillators' phases converge synchronously to 0 at different times. Our goal is to inquire into the nature of this phase convergence in the case of finite coupling. We seek for the relation between the coupling, the heterogeneity of the oscillators, and the connectivity pattern. We ask: What is the relation between these three characteristics that enhances phase condensation?

[20] We follow the line of exploration suggested by *Gong and van Leeuwen* [2004], analyzing a network model whose parameters are adapted so that the mutual phase convergence of individual nodes is enhanced. Specifically, following *Gleiser and Zanette* [2006], we set up a rewiring mechanism between the oscillators (active cells in thunderstorms) that is driven by the goal of achieving phase coherence. A random set of links between the oscillators is initially set up. Periodically, the network is searched for possible rewiring of the links. For every oscillator i we search for the oscillator j with the smallest phase difference $|\varphi_i - \varphi_j|$. If j is not in Post(i), then it is inserted into Post(i), replacing that oscillator k in Post(i) that has the largest phase difference $|\varphi_i - \varphi_k|$; Otherwise, nothing is done and the system remains

unchanged. The mechanism is controlled by two parameters, p, and μ. Here p is the probability of setting up the initial link between any pair of oscillators. If C time units is a nominal lifetime of an oscillator, then the rewiring mechanism will take place every μC time units.

2.4. Model Parameters

[21] The model parameters are then as follows. N is the number of oscillators active in the thunderstorm cells. In principal, each EAC may have several such oscillators and the number should vary between clouds. C is a nominal lifetime of noninteractive oscillator. In nature, that would be the time difference between successive flashes for a single isolated thunderstorm. It is clear that the charging rates in various parts of thunderclouds are different and depend on their dynamical and microphysical properties and their stage in the cloud life cycle. To take this into account, we associate to each oscillator i a parameter Δ_i, between 0.5 and 1, such that its lifetime with no interaction is $T_i = C/\Delta_i$. γ_i are the leakage rate factors, τ is the signal delay time, ε is the coupling strength factor, R_{ij} are the distances between oscillators i and j, p is the probability for setting up an initial connection between any pair of oscillators, and μ is the rewiring period.

3. Simulation

3.1. Initialization

[22] We use a discreet simulation to study the temporal evolution of the EAC network. Initially, the N electrically active cells are randomly distributed in a N $\times$ N grid of cells. EAC i is placed at coordinates (x_i, y_i), where x_i, y_i are integers. Next, distances R_{ij} between all pairs of EACs are calculated. Then the initial phases $\varphi_i(0)$ of each of the oscillators are drawn from a uniform distribution of values between 0 to 1, and the values of Δ_i are drawn from a uniform distribution of values between 0.5 and 1. Different Δ_i cause the oscillators to change their intensities (without interaction) in different rates, reflecting the natural variability between thunderstorm charging rates. Finally, the initial wiring scheme is constructed: every pair of oscillators (thunderstorms) is connected with connection probability p. A pair (i, j) of connected oscillators reflects the fact that they are mutually influencing each other (i belong to Pre(j) and j belongs to Pre(i)). In this simulation the sets of oscillator i - Pre(i) and Post(i) - are identical, for all values of i.

3.2. Simulation Step

[23] Each time step n, n = 1, 2, ... MC, consists of two substeps: phase advancements and rewiring (M is an integer that determines the simulation time).

3.2.1. Phase Advancements

[24] First, the new phase of each EAC, say, i, is advanced to $\varphi_i(n) = \varphi_i(n-1) + \Delta_i/C$. If $\varphi_i(n) \geq 1$, the EAC fires a lightning flash, the signal is added to the list of pending signals, and the phase of the EAC is set to $\varphi_i(n) = 0$. Next, we check if any pending signals are due to arrive at that time step (i.e., they were fired τ time units ago); in that case, the receiving EAC, i, changes its phase according to equations (9a) and (9b), where j is the sending EAC, such that $\varepsilon_{ij} = \varepsilon/R_{ij}$ and R_{ij} is the distance between i and j. The change in the intensity of the EAC is a combination of its natural intrinsic growth and the contribution from all other electrically connected active cells that had fired before that time step.

3.2.2. Adaptive Rewiring

[25] If n is an integer multiple of μC, the network has the potential to be rewired. For each EAC *i* we search for the EAC *j* with the closest phase to that of *i*; if *i* and *j* are already connected, no action is taken. Otherwise, we find the EAC k with the greatest phase difference from i, then replace the connection between *i* and *k* by a new connection between *i* and *j* thus removing the EAC k from the list of connected neighbors. In other words, we replace the least influencing link by the most influencing link [*Gleiser and Zanette*, 2006].

4. Results

[26] In this section we present the results of the simulation with the following parameters: N = 20; $\gamma = 1$; $\tau = 0.1$, C = 100, M = 50, $\mu = 0.5 - 4$ (step 0.5); $\varepsilon = 0.1–0.9$ (step 0.1); p = 0.1–0.9 (step 0.1). This simulation was not intended to exactly replicate the storm described in section 1.2 but rather to show how a network of thunderstorms each containing a number of LIF oscillators can explain the non-randomness of lightning activity that we observed. The model was tested for N = 20, a number that represents the existence of several charged regions within each of a few thundercloud cells, which is more typical of winter thunderstorms in the eastern Mediterranean and can be related to the Lebanon case study described above. We also tested the model for N = 200, a scale which would be more appropriate for describing the observed lightning activity in summer continental MCSs in the United States. In the case where the oscillators are completely independent, with random initiation times and random lifetimes, the null hypothesis would be that for large enough time intervals their phases would be evenly distributed. The frequency of occurrence of $\varphi = 1$ (namely, a firing, or lighting, event) will be equal to the frequency of occurrence of any other value. One way to estimate the level of phase convergence is by evaluating the difference between the observed values in the simulation and the predicted values by the null hypothesis. Let E(t) be the expected number of oscillators with $\varphi = 1$, assuming uniform distribution of phases, averaged over some time interval, t, and let O(t) be the observed number of oscillators with $\varphi = 1$ during that same time interval. As a measure for phase convergence, or synchronization, we use the standard χ^2 test [*D'Agostino and Stephens*, 1986] defined by

$$\chi^2(t) = (O(t) - E(t))^2/E(t). \tag{10}$$

Figures 4a and 4b present the values of χ^2 as functions of p and ε, respectively, for the time interval n = 300–400 time steps, in the case where $\mu = 0.5$. The critical value for $\chi^2(3,0.05)$ is 7.815; the values in our simulation are much larger, indicating that the distribution of intensities is far from uniform and should exhibit grouping and synchronicity. We also observe that there are critical values for ε and p in which the convergence jumps by almost an order of magnitude. These critical values are inversely dependent, suggesting it may be more useful looking at the χ^2 as a

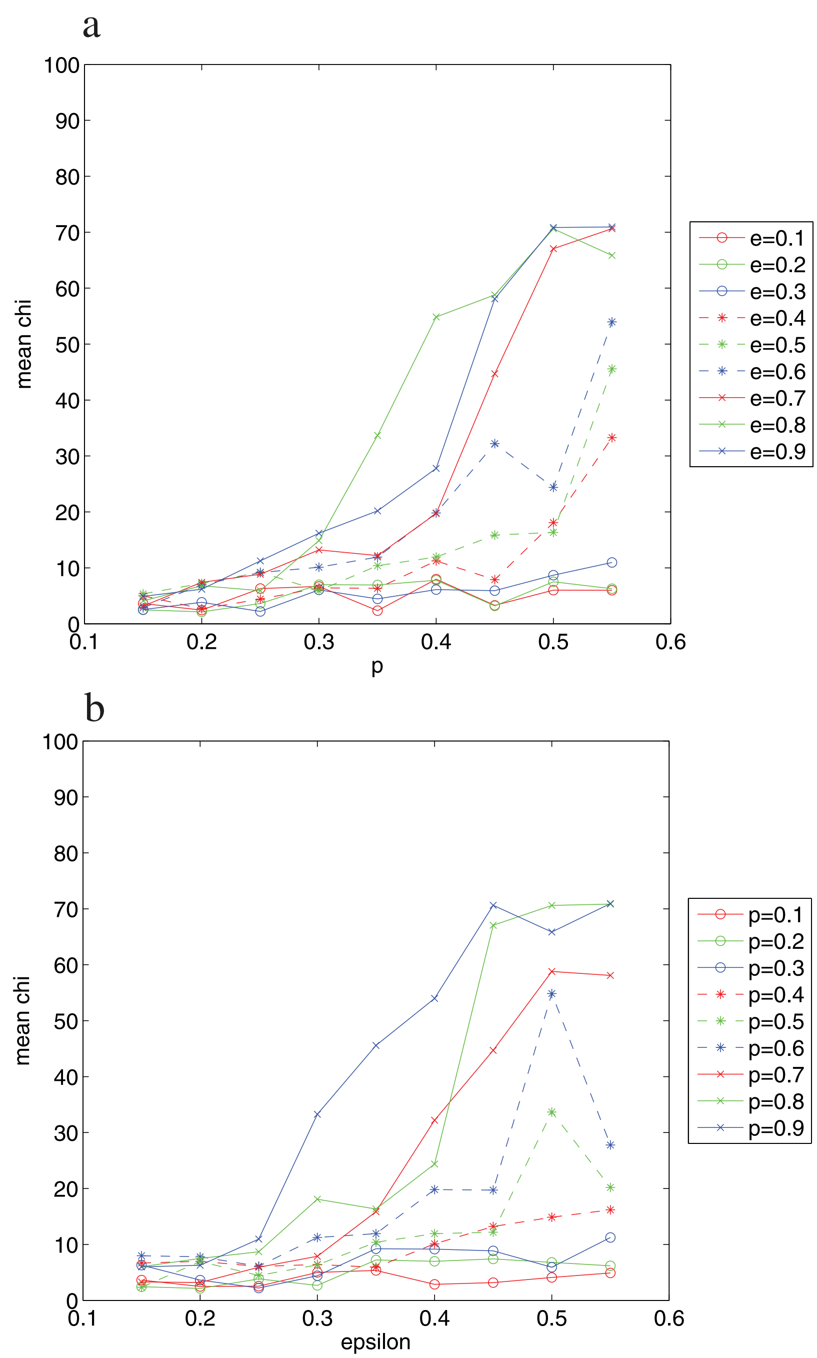

Figure 4. (a, b) Model results for N = 20 leaky integrate-and-fire oscillators. The values of χ^2 as functions of p (Figure 4a) and ε (Figure 4b) are shown, for the time interval n = 300–400 time steps, in the case where $\mu = 0.5$.

function of the group coupling g = $\varepsilon \cdot$ p. In Figure 5a we can observe a critical value of the group coupling g where there is a jump in the value of the convergence by an order of magnitude, signifying a change from an incoherent state to a more synchronized one. In other words, even for low values of coupling strength ε between the electrically active cells, the system can move to a coherent state, provided that the group of neighbors is large enough. We conducted several numerical experiments with varying parameters and sampled the network behavior at various times. The results for N = 200 are presented in Figure 5b and show the transition to a synchronized state more clearly.

5. Conclusion

[27] Our analysis started from the observation that lightning flashes tend to converge and cluster in rather short bursts with small time intervals. Effectively they synchronize their phases. We then developed an adaptive model of network of LIF oscillators that can explain this phenomenon. Indeed, *Mirollo and Strogatz* [1990] speculated that any

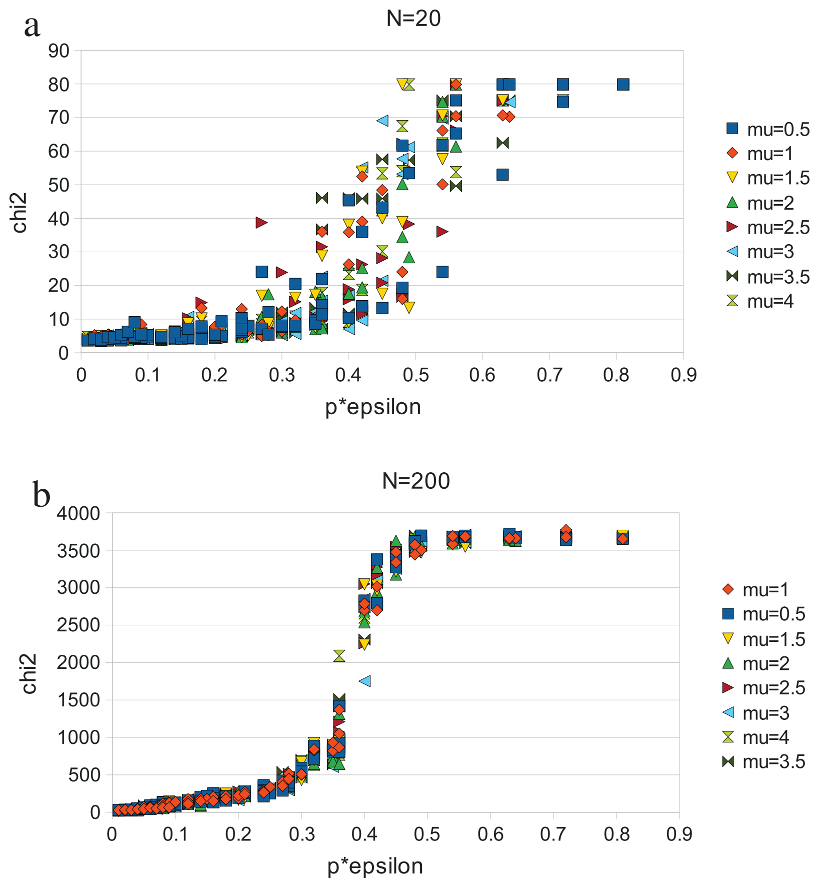

Figure 5. (a) The χ^2 as a function of the group coupling $g = \varepsilon \cdot p$ for various values of μ for N = 20. (b) The same as Figure 5a but for N = 200.

system of identical oscillators would end up firing in unison for almost any set of initial conditions, no matter how they were interconnected (provided these interconnections form a connected graph). For a system of nonidentical oscillators, they stated that "almost nothing is known for the case of pulse coupling." Our simulation results confirm the speculation of *Mirollo and Strogatz* [1990] and show that in a group of nonidentical oscillators which form a connected graph (not necessarily a fully connected system where each node is connected to all the others), the system will reach synchronicity, provided it evolves with an appropriate adaptive algorithm, for values of group coupling that exceed a certain threshold value.

[28] Coming back to the natural arena of active thunderstorms in a mesoscale convective system (MCS), we can hardly expect a situation of 200 EACs firing simultaneously (N was 200 in one of our simulations; see section 4). Typically, the population of mature cumulonimbus clouds that generate lightning flashes numbers in tens or fewer at any given time, even in squall lines and hurricanes. Nevertheless, the principle observation of synchronous firing by different cells, sometimes far from each other and certainly unconnected by long discharge channels, remains to be explained. One approach is based on the propagation of the electromagnetic field following a cloud-to-ground lightning flash. This approach calculates the far (radiation) field produced at any altitude by a lightning return stroke [*Shao et al.*, 2005; *Thottappillil et al.*, 2007]. Since the propagation speed is c, the time between the flash and the effect felt at a radius of 100 km is measured in microseconds. The amplitude of the far field in such a range is a few V m^{-1}, and it decays in ~10 ms. While this seems minute compared to the conventional and even runaway breakdown electric fields, it may still be a decisive addition in subcritical regions, where the in-cloud electric field is just below avalanche values. *Cooray* [2003b] gives an overview of mathematical modeling of return strokes, where calculations by C. A. Nucci et al. (Lightning induced voltages on overhead lines, paper presented at Ninth International Symposium on Lightning Protection, Association of Austrian Electrical Engineering, Graz, Austria, 1988) and V. A. Rakov and A. A. Dulzon (A modified transmission line model for lightning return stroke field calculations, paper presented at Ninth International Symposium on EMC, ETH Zentrum-IKT, Zurich, Switzerland, 1991) show that in close ranges of 2–5 km from the vertical return stroke current (simulated by the well-known transmission line model and its variations), induces electric field changes on the order of 500 and 80 V m^{-1}, respectively, on time scales of 100 μs. This close-range effect is strong enough to affect other charged regions within the same cell and expedite their approach to breakdown. This may explain the non-Poisson nature of consecutive flashes we described in section 1.3 and matches well our model of ensembles of LIF oscillators.

[29] In a recent paper, *Ondrášková et al.* [2008] quote Nikola Tesla in stating that electromagnetic waves may trigger discharges. They suggest that lightning-induced electromagnetic waves in the Schumann Resonance band of a few Hz, orbiting the Earth within the surface-ionosphere waveguide, might trigger lightning discharges. If indeed this process is effective it will complement the direct addition by the far field. Successive flashes may thus cause a significant enhancement which will join the already operating charge-separation mechanisms in a certain cloud, and drive it toward breakdown. The time delays between successive flashes in the storms will thus reflect the interdependence of lightning occurrence. The synchronicity of lightning flashes in separate and distant cells is most likely a transient feature in many multicell storms. The dominant factors that reflect the strength of coupling between the various cells would be their respective distances and the individual microphysical conditions in each member of the network, which define its internal charging and flash rate. The next step in this research would be to determine which natural factors can be represented by the group-coupling term that determines the threshold between the coherent and random states of the system.

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